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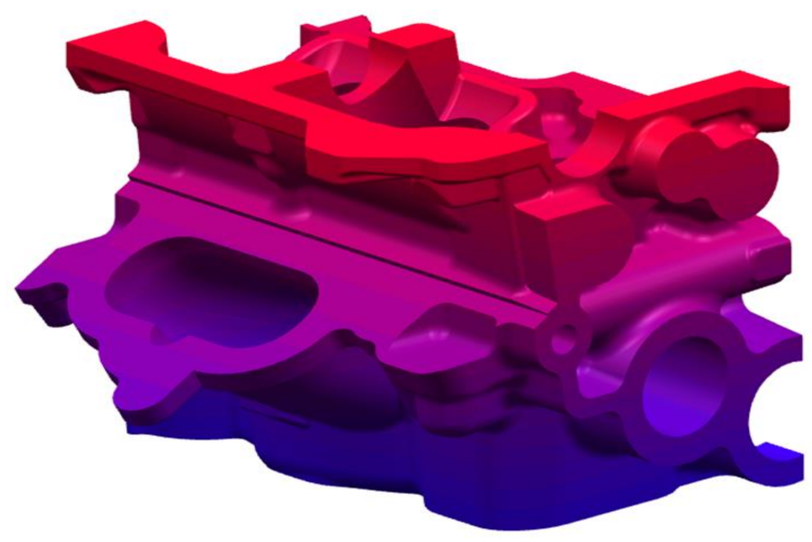

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Computing Mass Properties of Objects with Continuously Varying Density Distributions

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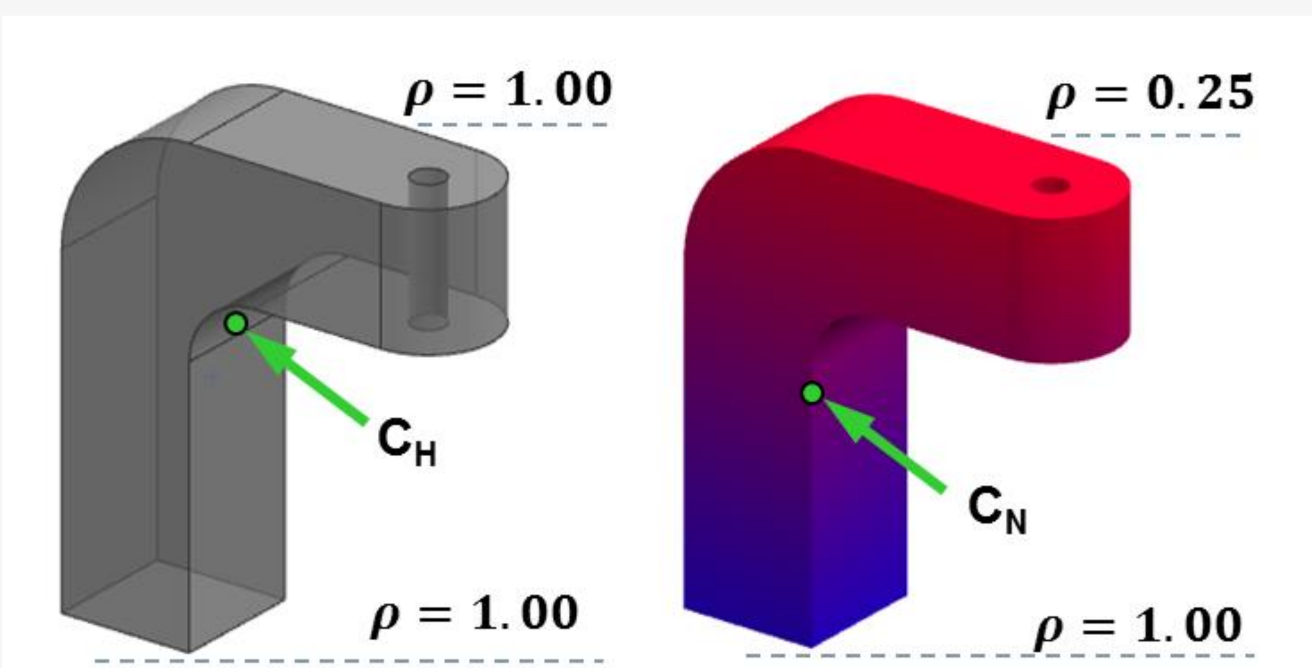
Additive manufacturing enables continuously varying material density distributions

Leverage material density gradation in design



Constant density Part unstable	Graded density Improved stability

Mass properties are required to compute static and dynamic behavior of objects

Mass properties are volume integrals over object S
Existing approaches assume density ρ = constant

Volume: $v = \int_S dxdydz$ Mass: $m = \int_S \rho \, dxdydz$

Center of mass:

$$C = \frac{1}{m}\left(\int_S x\rho \, dxdydz\,, \int_S y\rho dxdydz\,, \int_S z\rho dxdydz\right)$$

Moments of Inertia: $I^O = \begin{bmatrix} I^O_{xx} & -I^O_{xy} & -I^O_{xz} \\ -I^O_{yx} & I^O_{yy} & -I^O_{yz} \\ -I^O_{zx} & -I^O_{zy} & I^O_{zz} \end{bmatrix}$

$$I^O_{xx} = \int_S (y^2+z^2)\rho dxdydz \qquad I^O_{xy} = I^O_{yx} = \int_S xy\rho dxdydz$$

$$I^O_{yy} = \int_S (z^2+x^2)\rho dxdydz \qquad I^O_{xz} = I^O_{zx} = \int_S xz\rho dxdydz$$

$$I^O_{zz} = \int_S (x^2+y^2)\rho dxdydz \qquad I^O_{yz} = I^O_{zy} = \int_S yz\rho dxdydz$$

Proposed Method: Adaptive numerical integration algorithm using Shunn-Ham Quadrature Rules

1. Triangulate object S surface. Pick reference point Q.
2. For each triangle ABC, construct tetrahedron T with vertices QABC.
3. Compute mass properties of tetrahedron T
   a. If integrand is polynomial (degree d ≤ p), compute exact integral using Shunn-Ham precision 'd' rule
   $$\int_T f \, dxdydz = v \sum_{j=1}^{m} w_j f(N_j)$$
   b. Else, compute integral using Shunn-Ham precision 'p' rule with adaptive subdivision of tetrahedron
   $$\int_T f \, dxdydz \approx v \sum_{j=1}^{m} w_j f(N_j)$$
4. Aggregate mass properties of tetrahedra

$$\int_S f \, dxdydz \approx \sum_{i=1}^{n} \int_{T_i} f \, dxdydz$$

Validation on Unit Tetrahedron

Mass property	Proposed Method (Quartic Precision)	Actual (Mathematica) $\rho = 1 + x^2 + 2y^2 + 3z^2$
m	0.266666	4/15 = 0.266666
(C_x, C_y, C_z)	(0.239583, 0.260416, 0.281250)	(23/96 = 0.239583, 25/96 = 0.260416, 9/32 = 0.28125)
I_{xx}; I_{yy}; I_{zz}	0.281250; 0.029365; 0.033333	8/315 = 0.025396; 37/1260 = 0.029365; 1/30 = 0.033333
I_{xy}; I_{yz}; I_{zx}	0.013095; 0.014682; 0.013888	11/840 = 0.013095; 37/2520 = 0.014682; 1/72 = 0.013888

Mass property	Proposed Method (Quartic Precision)	Actual (Mathematica) $\rho = \cos(x)\cos(y)\cos(z)$
m	0.142829976	(1/8)(-Cos[1]+2Sin[1]) = 0.142829957
$(C_x$, C_y, $C_z)$	(0.245475760, 0.245475760, 0.245475760	$C_x = C_y = C_z$ Sin[1]/(3 (-Cos[1]+2 Sin[1])) = 0.245475749
I_{xx} I_{yy} I_{zz}	0.013653457 0.013653457 0.013653458	$I_{xx} = I_{yy} = I_{zz}$ 1/48 (9 Cos[1]-5 Sin[1]) = 0.013653454
I_{xy} I_{yz} I_{zx}	0.007014365 0.007014365 0.007014365	$I_{xy} = I_{yz} = I_{zx}$ 1/96 (-19 Cos[1]+13 Sin[1]) = 0.007014364