Averaging compatible surfaces for free-form tolerancing

“How to improve your manufacturing accuracy / reliability?”

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Overview

▶ Inconsistent output ≠ Consistently bad output.
▶ There are consistent & inconsistent machining errors.
▶ Analyzing average error and error variability separately facilitates decision making.
▶ Nominal ≠ Average : Better plan?
▶ Variability > Maximum : Better machine?

Computing average and variability

▶ Input: Nominal shape \( N \), machined shapes \( M_i \).
▶ Output: Average mesh, variability.

Algorithm:
▶ Sample \( M_i \) to obtain machined shapes point clouds.
▶ Filter and register point clouds.
▶ Triangulate each point cloud. item Compute average triangle mesh and correspondences:

\[
\text{Input} \quad \text{Candidate mesh, Translated point clouds} \quad \text{Triangle mesh averaging algorithm} \quad \text{Average mesh} \quad \text{Output} \quad \text{Average mesh + Correspondences}
\]

▶ Compute variability from correspondences.

Results

▶ Comparison with gradient descent:
▶ Snap iteration converges in 2-3 steps.
▶ Snap iterations produce low distortions.

▶ Snap:

Salient details

Compatible smooth manifolds and the Valley Average

▶ Set of embedded manifolds compatible if pairwise closest projection maps are homeomorphisms.
▶ A compatible set has a natural average - the valley average - a connected subset of points that lie on the valley of the sum of squared distance field \( Q \).

Averaging planes

▶ Planes compatible when hessian of \( Q \) is a positive definite quadratic form.
▶ Valley average of a set of planes is a plane.
▶ Coincident planes:
  ▶ Average is coincident.
  ▶ Maximizes sum of unit circle projection areas.
▶ Non-coincident planes:
  ▶ Average directed as coincident case
  ▶ Incident to point minimizing \( Q \).
▶ Numerically robust formula to project on average.

Snap iterations

▶ A snap iteration comprises:
▶ Closest project: Given candidate point \( p \), compute closest projection to \( i \)th mesh \( p_i \).
▶ Average computation: Compute average plane \( P \) using tangent plane approximation \( P_i \) at \( p_i \).
▶ Projection on average plane: Project \( p \) onto \( P \).