

# Averaging compatible surfaces for free-form tolerancing

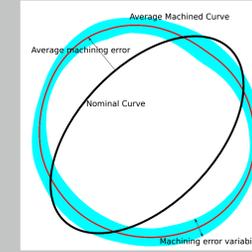
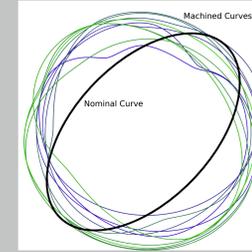
"How to improve your manufacturing accuracy / reliability?"

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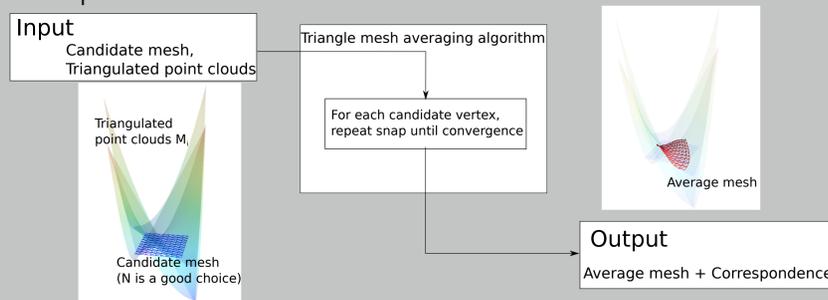
## Overview

- ▶ Inconsistent output  $\neq$  Consistently bad output.
  - ▷ There are consistent & inconsistent machining errors.
- ▶ Analyzing average error and error variability separately facilitates decision making.
  - ▷ Nominal  $\neq$  Average : Better plan?
  - ▷ Variability  $>$  Maximum : Better machine?



## Computing average and variability

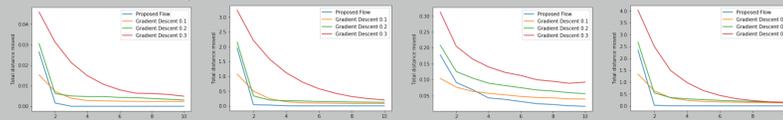
- ▶ Input: Nominal shape  $N$ , machined shapes  $M_i$ .
- ▶ Output: Average mesh, variability.
- ▶ Algorithm:
  - ▷ Sample  $M_i$  to obtain machined shapes point clouds.
  - ▷ Filter and register point clouds.
  - ▷ Triangulate each point cloud. :item Compute average triangle mesh and correspondences:



- ▷ Compute variability from correspondences.

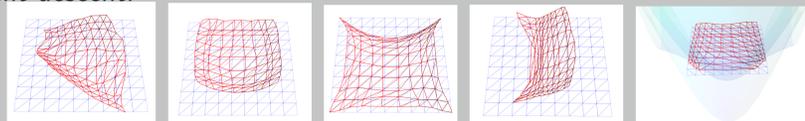
## Results

- ▶ Comparison with gradient descent:
  - ▷ Snap iteration converges in 2-3 steps.

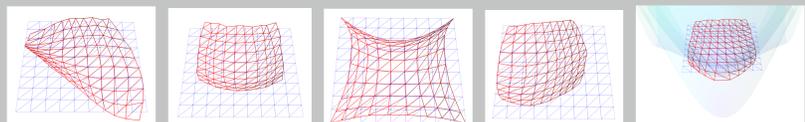


- ▷ Snap iterations produce low distortions.

- ▶ Gradient descent:



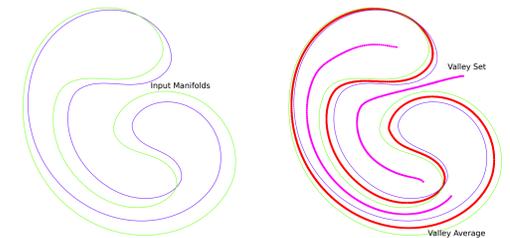
- ▶ Snap:



## Salient details

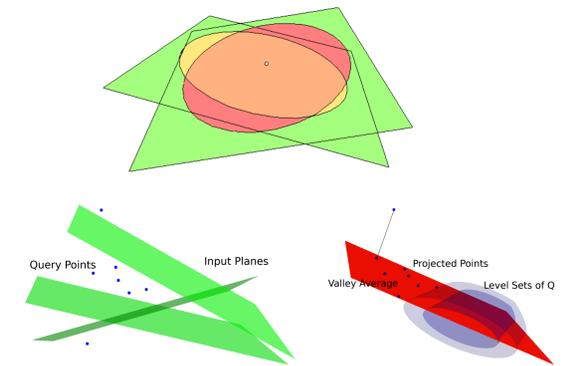
### Compatible smooth manifolds and the Valley Average

- ▶ Set of embedded manifolds compatible if pairwise closest projection maps are homeomorphisms.
- ▶ A compatible set has a natural average - the valley average - a connected subset of points that lie on the valley of the sum of squared distance field  $Q$ .



### Averaging planes

- ▶ Planes compatible when hessian of  $Q$  is a positive definite quadratic form.
- ▶ Valley average of a set of planes is a plane.
  - ▷ Coincident planes:
    - ▶ Average is coincident.
    - ▶ Maximizes sum of unit circle projection areas.
  - ▷ Non-coincident planes:
    - ▶ Average directed as coincident case
    - ▶ Incident to point minimizing  $Q$ .
- ▶ Numerically robust formula to project on average.



### Snap iterations

- ▶ A snap iteration comprises:
  - ▷ Closest project: Given candidate point  $p$ , compute closest projection to  $i^{th}$  mesh  $p_i$ .
  - ▷ Average computation: Compute average plane  $P$  using tangent plane approximation  $P_i$  at  $p_i$ .
  - ▷ Projection on average plane: Project  $p$  onto  $P$ .

