Towards Thick Shells with Isostatic Microstructure

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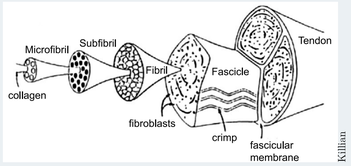
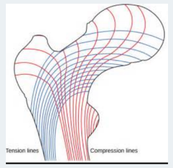
Motivation

- Manufacturing at micro-scale requires accurate **material structure representation** in order to predict physical properties
- Therefore a variety of constrained, isostatic, irregular micro-structures need to be accurately and efficiently represented
- Self-similar and foliated (layered) structures support design across scales

Inspiration from Nature

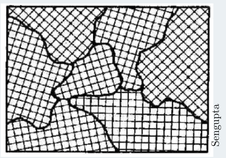


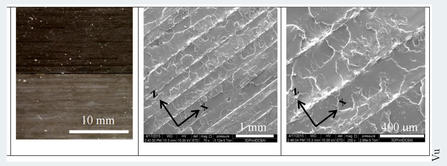
Killian



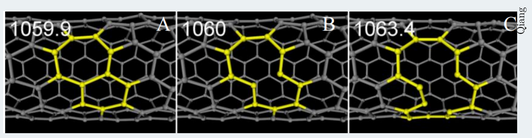
Analysis

Classical Finite Element Analysis is not enough!


Sengupta



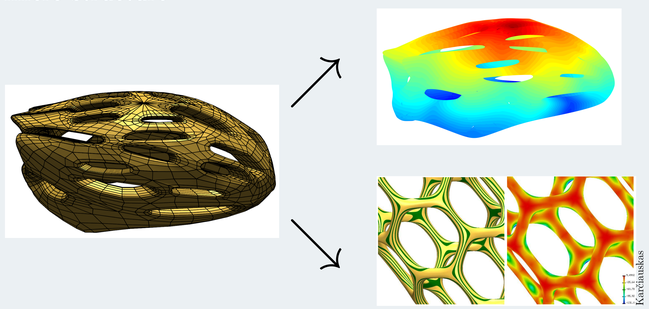
Vu



Qiang

Strain deformation

Thin shell vs micro-structure


Karčiauskas

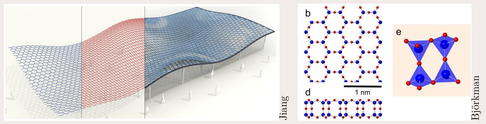
Approach

- **Input:** Coarse macro-geometry with physical characteristics and response
- **Output:** 3D framework of interconnected 2D isostatic layers suitable for manipulating , analyzing, and optimizing design of physical characteristics
- **Steps:**
  - Fine macro-geometry
  - Foliation of enclosed solid.
  - Layers conforming to the foliation
  - Realistic fabricated/assembled material micro-structure
  - Provably complete generators for all irregular isostatic frameworks
  - Self-similarity and repetition for hierarchical representation
  - Gluing to boundary or neighboring regions while preserving isostaticity

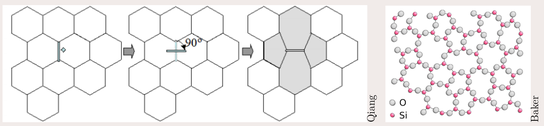
Foliation of solid


Nguyen
Campen

Mapping micro-structure onto and stacking foliation layers


Jiang
Björkman

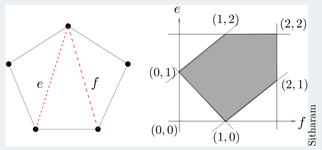
Generating irregular isostatic frameworks



Qiang
Baker

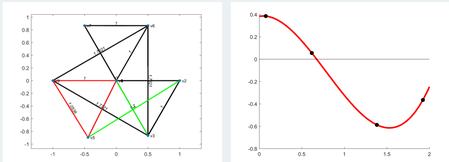
Results

Distance defines isostatic micro-structure (realizing the structure)
Algorithm for finding a convex search space

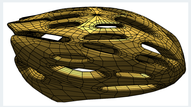


Sitharam

Algorithm for constructing the structure



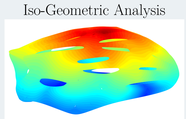
Results Continued



Control Mesh

C^1 Surface

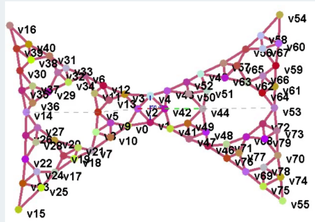
Iso-Geometric Analysis



Geodesics

Macro

Micro



A graph G with given edge lengths can be realized efficiently if there is a sequence of (non-edge, edge) pairs $(a_1, d_1), (a_2, d_2), ...(a_k, d_k)$ such that

- $G \setminus \{d_1, d_1, ...d_k\}$ has a convex search space over $\{a_1, a_1, ...a_k\}$
- $G \cup \{a_1, a_1, ...a_k\}$ can be constructed one vertex at a time

Theorem:
If G is planar, such a sequence exists with elimination order 5.
(d_i depends on at most five a_j's, $j > i$)

References

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Acknowledgements

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