

# DTHB3D\_Reg: Dynamic Truncated Hierarchical B-Spline Based 3D Nonrigid Image Registration



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## Abstract

We present a robust approach to perform 3D nonrigid image registration suitable for large deformation and topology change, and develop a software package named DTHB3D\_Reg (Dynamic Truncated Hierarchical B-spline based 3D image Registration). The optimum spatial transformation, defined using truncated hierarchical B-splines, is obtained through the minimization of an energy functional. Control points are dynamically updated without constructing large matrices as in finite element method. To improve the computational efficiency, an adaptive strategy carries out refinement only in the regions with large deformation. We can achieve change in topology of the images and preserve sharp features, thus improving the accuracy of the registration process

## Registration Framework

- Spatial transformation function  $f(x): I_1(x) \rightarrow I_2(x)$
- Tri-quadratic B-splines create a  $C^1$ -continuous mapping in  $R^3$  space

$$f(x) = \sum_{k=1}^{N_b} P_k \phi_k(x)$$

- where  $P_k$  is a set of control points associated with the trivariate basis functions  $\phi_k(x)$ .

### Energy functional:

$$E(f(x)) = \int_{\Omega} g(x) (I_2(x) - I_1(f(x)))^2 d\Omega + \lambda_1 \int_{\Omega} (\|f_{,\varepsilon}(x)\|_2^2 + \|f_{,\eta}(x)\|_2^2 + \|f_{,\zeta}(x)\|_2^2) d\Omega + \lambda_2 \int_{\Omega} (\|f_{,\varepsilon}(x)\|_2^2 \|f_{,\eta}(x)\|_2^2 - (\langle f_{,\varepsilon}(x), f_{,\eta}(x) \rangle)^2 + \|f_{,\eta}(x)\|_2^2 \|f_{,\zeta}(x)\|_2^2 - (\langle f_{,\eta}(x), f_{,\zeta}(x) \rangle)^2 + \|f_{,\zeta}(x)\|_2^2 \|f_{,\varepsilon}(x)\|_2^2 - (\langle f_{,\zeta}(x), f_{,\varepsilon}(x) \rangle)^2) d\Omega$$

FIDELITY TERM

SECOND ORDER REGULARIZATION

FIRST ORDER REGULARIZATION

- Formulation without matrix assembly:  $\frac{P_i^{t+1} - P_i^t}{\epsilon} = -\delta E_i(f(x))$

- Adaptive refinement using THB splines:

$$I_g = \|\nabla(I_1^{-1}(f(x)) - I_2(x))\|$$

$G_{mean}$  is the average value of  $I_g$ .

For all the active B-spline basis functions at current level find the average of the  $I_g$  values denoted as  $G_j$ .

Construct hierarchical basis functions and control points. Update control points via optimization.

If  $G_j > \rho * G_{mean}$ , REFINE

## Results

### SYNTHETIC IMAGES:

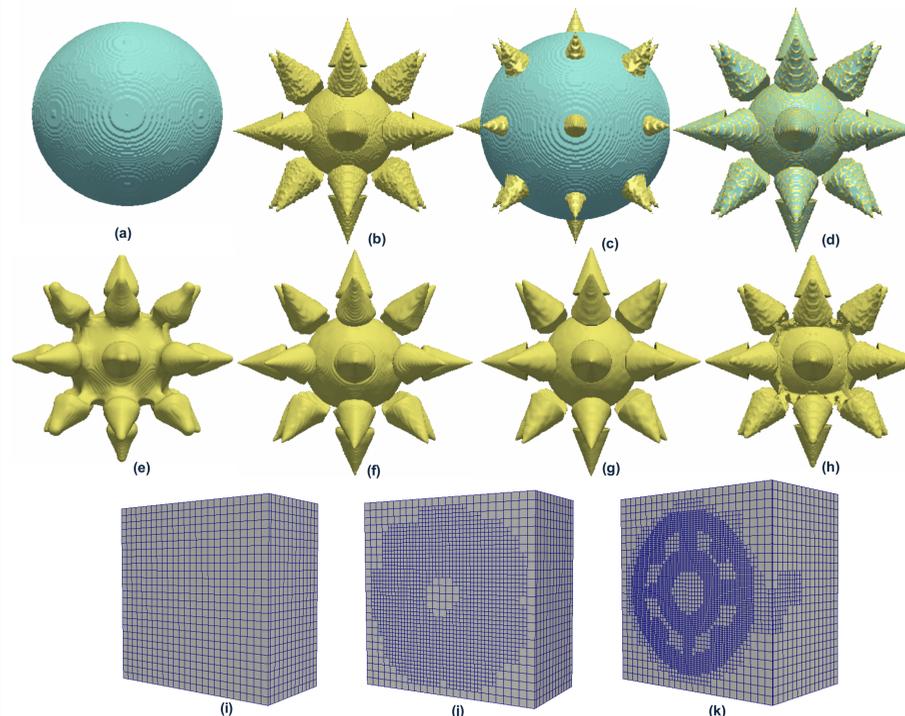


Fig 1: Sphere to Sun, the source image (a), the target image (b), initial image difference (c), final image difference (d), evolving image at level 1 (e), evolving image at level 2 (f), final registered image using DTHB3D\_Reg (g) and final registered image using the level set method [2] (h). The adaptively refined control grids at each refinement level are shown in (i-k).

### MEDICAL IMAGES:

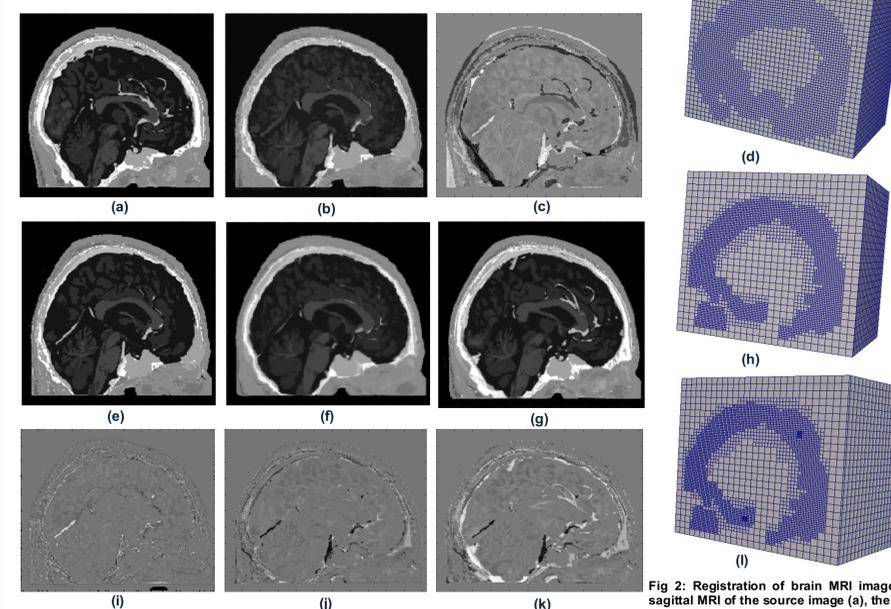
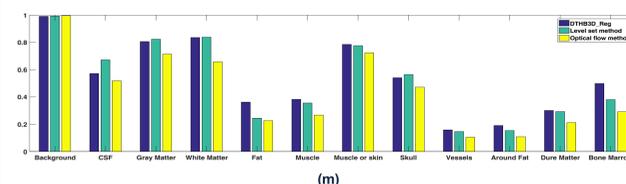
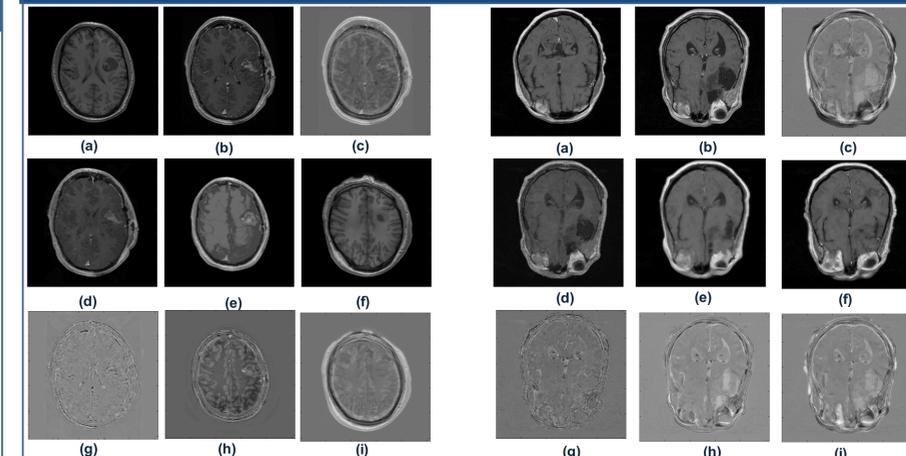


Fig 2: Registration of brain MRI image: the sagittal MRI of the source image (a), the target image (b) and the initial image difference (c), the registered image using DTHB3D\_Reg (d), the level set method (e) and the optical flow method (f), the final image difference between the registered and target images using DTHB3D\_Reg (g), the level set method (h) and the optical flow method (i). Fourth column: the adaptively refined grids at levels 2 (d), 3 (h) and 4 (l). Average segmentation accuracy between the registered images and the target images of 10 pairs of brain MRI evaluated using the DTHB3D\_Reg method, the level set method and the optical flow method, respectively (m).



## Results



Figs. 3 and 4: The axial MRI of the source image (a), the target image (b) and the initial image difference (c). Second row: the registered image using DTHB3D\_Reg (d), the level set method (e) and the optical flow method (f). Third row: the final image difference between the registered and target images using DTHB3D\_Reg (g), the level set method (h) and the optical flow method (i).

## Conclusion

- A robust method to perform 3D nonrigid image registration suitable for large image deformation and topology change is proposed.
- Adaptive refinement is introduced using THB-splines by locally detecting and refining only those regions that undergo fine-scale deformations.
- Dynamic implementation of the optimization is carried out where the energy functional is minimized using the strong formulation directly.
- Based on the numerical results, we have shown that our method is more accurate in capturing large deformation and is robust to topology change.

## References

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