Power histograms: a new approach for circle detection in images

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Introduction

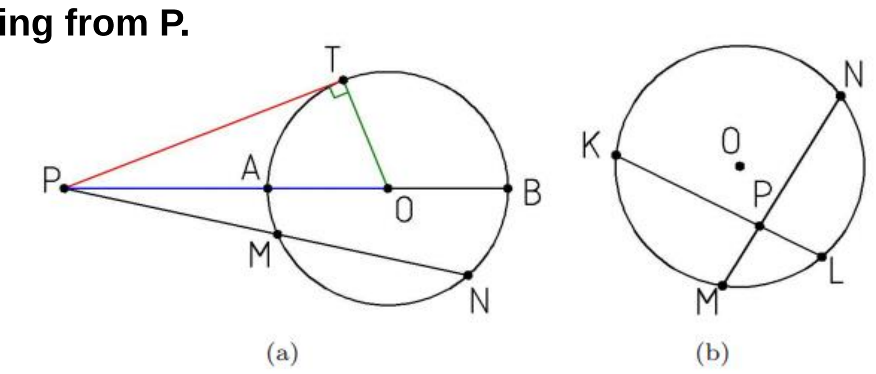
Automatic circle detection in digital images is of fundamental importance to pattern recognition and computer vision, particularly in applications such as product inspection and assembly, traffic sign detection, robot vision, pupil and iris localization, vectorization of hand-sketched drawings, to name just a few. An ideal circle detection algorithm should work with synthetic, natural, and noisy images, have real-time speed, and be able to detect multiple small and large circles with few or zero false detections.

Theoretic Fundamentals

- Circle Power Theorem
  - Given a Circle O with O being its center and r being its radius, the power of an arbitrary point related to O is a real number that reflects the relative distance of a given point from the circle. Specifically, the power of a point P with respect to a circle O is defined as

$$pow(\odot O, P) = \overline{OP}^2 - r^2$$

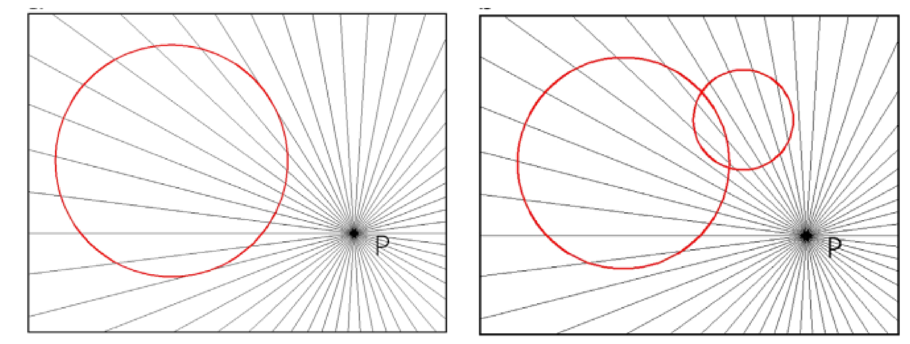
  - The power of point P can be defined equivalently as the product of distances from point P to the two intersection points of any ray emanating from P.



$$\overline{PT}^2 = \overline{PM} \times \overline{PN} = \overline{PA} \times \overline{PB} = pow(\odot O, P) \qquad \overline{PM} \times \overline{PN} = \overline{PK} \times \overline{PL} = -pow(\odot O, P)$$

Theoretic Fundamentals

- Power histogram of an image
- For image I, let E be the set of edge pixels in the image. Let P be a point in E, and R be a set of M rays emanating from P.



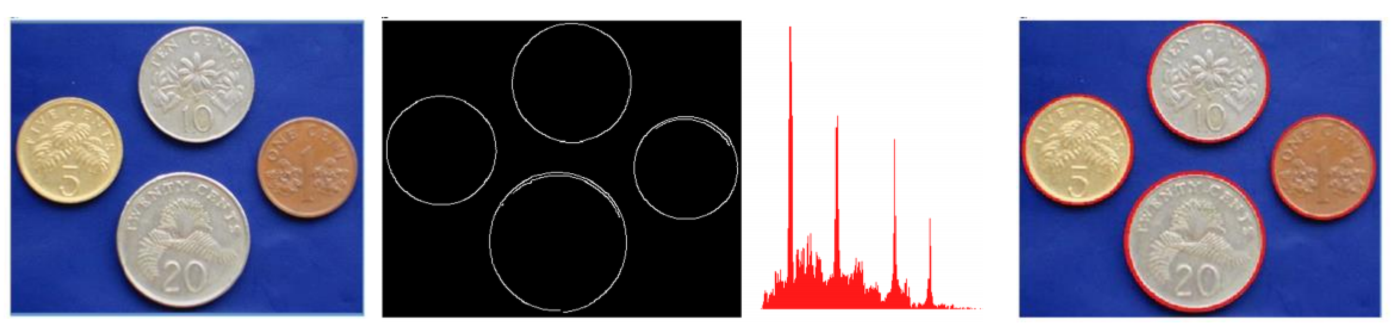
- By collecting all the distances of the edge pixels in E to reference point P, and calculating their pairwise distance products will give a power set P along one ray direction (note that the term “power” indicates the product of factors)

$$\mathcal{P}_j = \{\overline{Pe_{k1}} \times \overline{Pe_{k2}}, \quad \forall e_{k1}, e_{k2} \in E_j\}$$

- The power histogram of image I, denoted as PH, with respect to a point P is defined upon all the power sets calculated from the M rays emanated from reference point P

Proposed Method

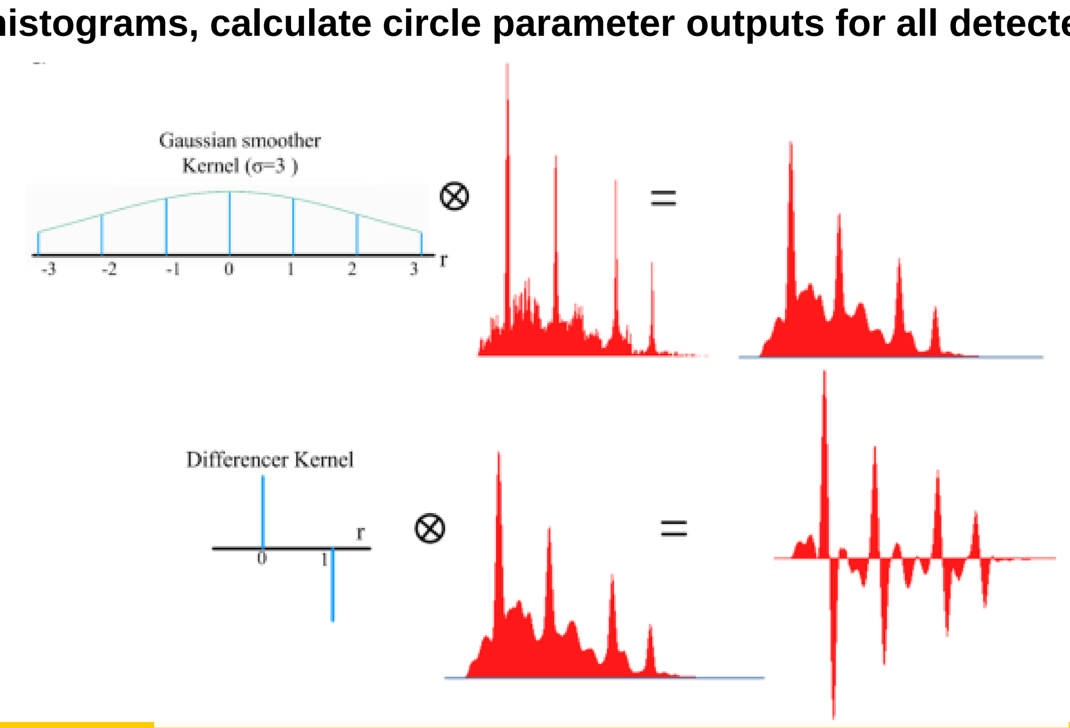
- Overview
  - Identify edge pixels within an input image
  - construct N power histograms from N random reference points
  - Detect peaks in each power histogram
  - For each detected peak, perform cross-validation with N power histograms, calculate circle parameter outputs for all detected circles



- Cross Validation
  - Identify edge pixels within an input image
  - construct N power histograms from N random reference points
  - Detect peaks in each power histogram
  - For each detected peak, perform cross-validation with N power histograms, calculate circle parameter outputs for all detected circles

Proposed Method

- Peak Detection
  - Identify edge pixels within an input image
  - construct N power histograms from N random reference points
  - Detect peaks in each power histogram
  - For each detected peak, perform cross-validation with N power histograms, calculate circle parameter outputs for all detected circles

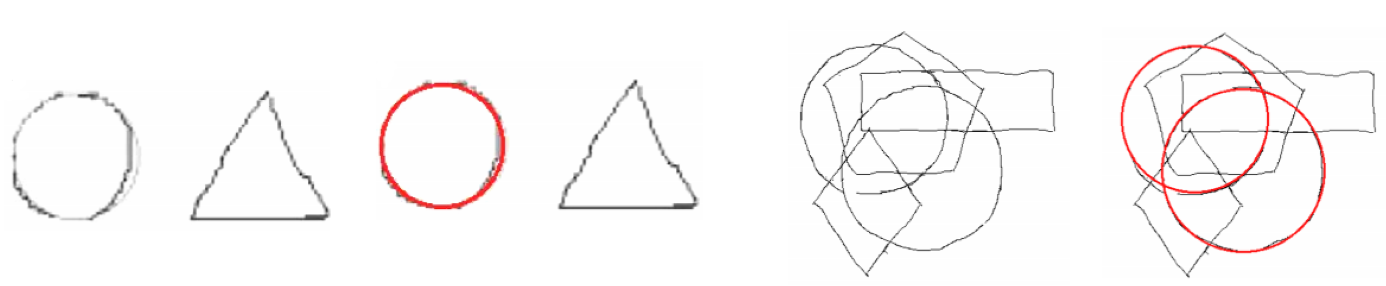


Experiment Results

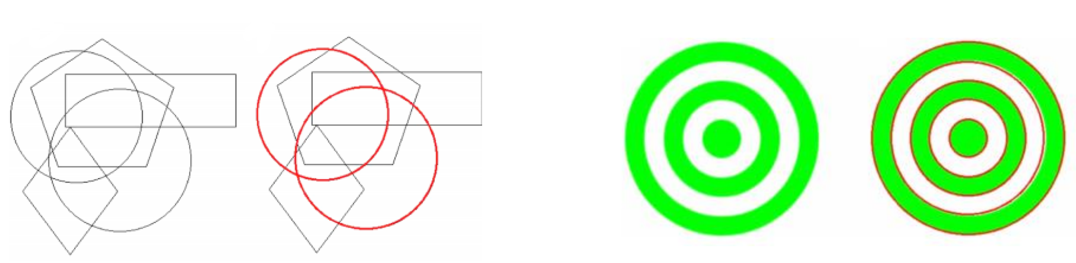
▪Experiments on Real-life Images

Experiment Results

▪Experiments on Hand-sketched Images

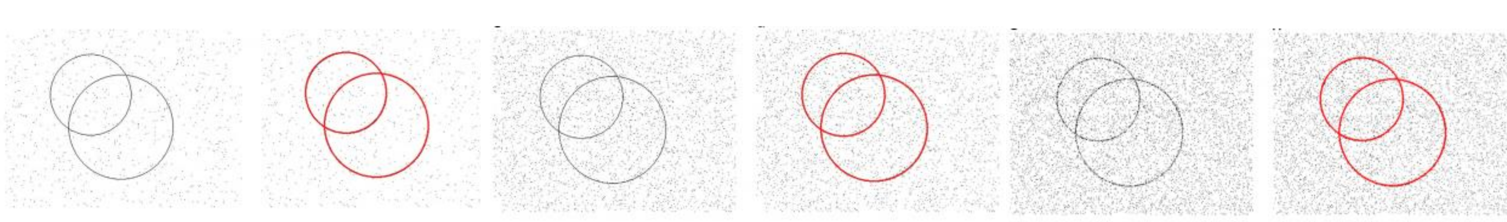


▪Experiments on Synthetic Images



Experiment Results

▪Experiments on Noisy Images



Noise level	CHT		RCD		PHCD
	cvHoughCircles parameters (dp[a], min_dis[b], thr[c])	Execution-time (ms)	Parameter (T_f[d])	Execution-time (ms)	Execution-time (ms)
0.01	1.4, 16, 80	63	10,000	38	305
0.02	1.4, 16, 80	78	10,000	42	624
0.03	1.4, 16, 90	99	10,000	47	283
0.04	1.4, 16, 100	110	10,000	63	290
0.05	1.8, 16, 160	172	10,000	76	298
0.06	1.8, 16, 190	198	50,000	200	333
0.07	1.8, 16, 210	254	100,000	354	366
0.08	1.8, 16, 230	397	150,000	652	390
0.09	1.8, 16, 260	672	200,000	890	418
0.10	1.8, 16, 280	1059	300,000	1267	450
0.11	1.8, 16, 310	1377	600,000	2538	485
0.12	1.8, 16, 330	1812	1,000,000	4330	523
0.13	2.1, 16, 470	2677	2,000,000	6195	576
0.14	2.0, 16, 460	3025	3,000,000	–	644
0.15	1.9, 16, 450	3875	5,000,000	–	710
0.15+[e]	2.1, 16, 450	> 10,000	5,000,000	–	912

[a] dp: The inverse ratio of resolution.
[b] min_dis: The minimum distance between circles.
[c] thr: The accumulator threshold.
[d] T_f: The number of tolerable failures.
[e] 0.15+: Perturbed image plus 15% added noise.

Conclusions

- The proposed algorithm for the automatic detection of circular shapes among cluttered and noisy images independent of the conventional Hough transform principles, or the RANSAC-based sampling and validation strategy.
- Based on the basic power theorem of circles, the proposed method transforms the edge pixels on an image into power histograms which denote the distribution of pairwise distance products among all intersection points along different rays.
- Using the peaks in the power histograms, the method can significantly reduce the search space for potential circles and improve detection accuracy.
- The presented method is capable of detecting circles accurately in synthetic images, and is robust under the presence of noise.
- The method can also reliably detect multiple circles in hand-drawn or real-life images, even when the circles are shown and present significant occlusions, discontinuities or incompleteness.

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