Power histograms: a new approach for circle detection in images
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Introduction
Automatic circle detection in digital images is of fundamental importance to pattern recognition and computer vision, particularly in applications such as product inspection and assembly, traffic sign detection, robot vision, pupil and iris localization, vectorization of hand-sketched drawings, to name just a few. An ideal circle detection algorithm should work with synthetic, natural, and noisy images, have real-time speed, and be able to detect multiple small and large circles with few or zero false detections.

Theoretic Fundamentals
- Circle Power Theorem
  - Given a Circle O with O being its center and r being its radius, the power of an arbitrary point related to O is a real number that reflects the relative distance of a given point from the circle. Specifically, the power of a point P with respect to a circle O is defined as
    \[ \text{power}(O, P) = OP^2 - r^2 \]
  - The power of point P can be defined equivalently as the product of distances from point P to the two intersection points of any ray emanating from P.

Proposed Method
- Overview
  - Identify edge pixels within an input image
  - Construct N power histograms from N random reference points
  - Detect peaks in each power histogram
    - For each detected peak, perform cross-validation with N power histograms, calculate circle parameter outputs for all detected circles

- Cross Validation
  - Identify edge pixels within an input image
  - Construct N power histograms from N random reference points
  - Detect peaks in each power histogram
    - For each detected peak, perform cross-validation with N power histograms, calculate circle parameter outputs for all detected circles

Experiment Results
- Experiments on Hand-sketched Images
- Experiments on Noisy Images
- Experiments on Synthetic Images

Experiment Results
- Experiments on Synthetic Images

Proposed Method
- Peak Detection
  - Identify edge pixels within an input image
  - Construct N power histograms from N random reference points
  - Detect peaks in each power histogram
    - For each detected peak, perform cross-validation with N power histograms, calculate circle parameter outputs for all detected circles

Experiment Results
- Experiments on Hand-sketched Images
- Experiments on Noisy Images
- Experiments on Synthetic Images

Experiment Results
- Experiments on Hand-sketched Images
- Experiments on Noisy Images
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Conclusions
- The proposed algorithm for the automatic detection of circular shapes among cluttered and noisy images independent of the conventional Hough transform principles, or the RANSAC-based sampling and validation strategy.
- Based on the basic power theorem of circles, the proposed method transforms the edge pixels on an image into power histograms which denote the distribution of pairwise distance products among all intersection points along different rays.
- Using the peaks in the power histograms, the method can significantly reduce the search space for potential circles and improve detection accuracy.
- The presented method is capable of detecting circles accurately in synthetic images, and is robust under the presence of noise.
- The method can also reliably detect multiple circles in hand-drawn or real-life images, even when the circles are shown and present significant occlusions, discontinuities or incompleteness.